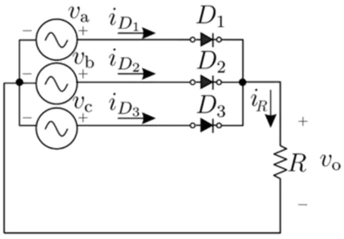
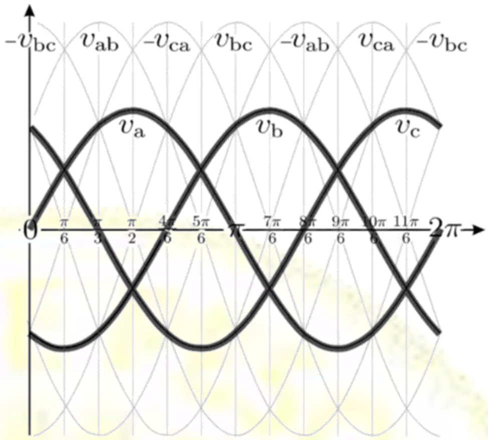
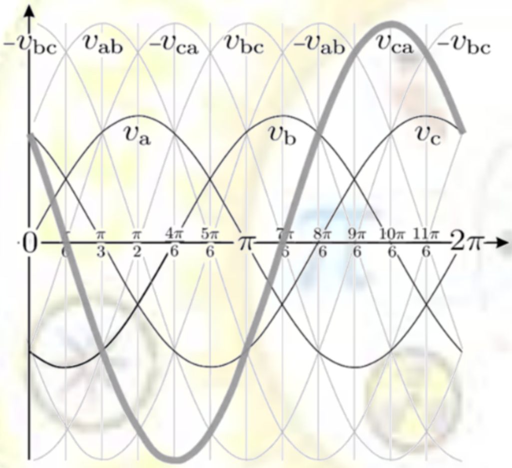
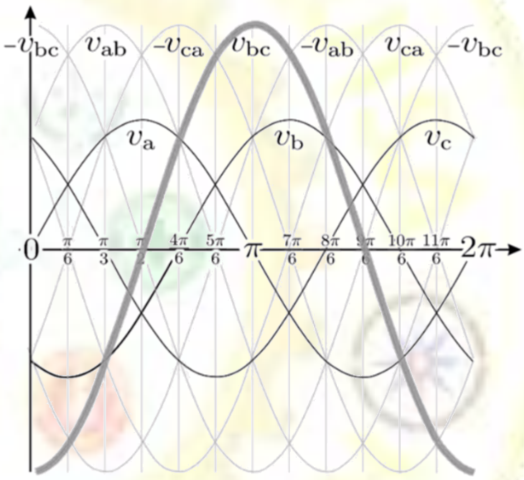
Relação entre as fases de 120^0

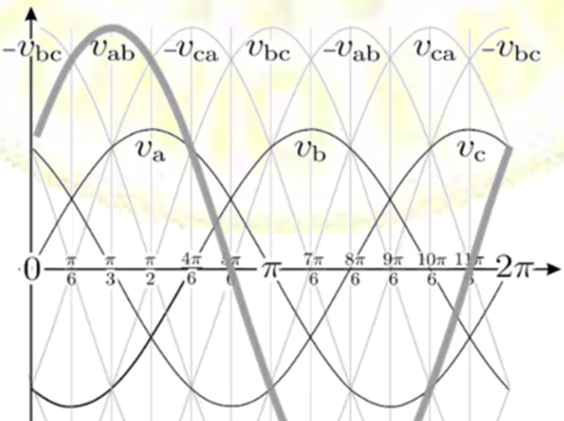


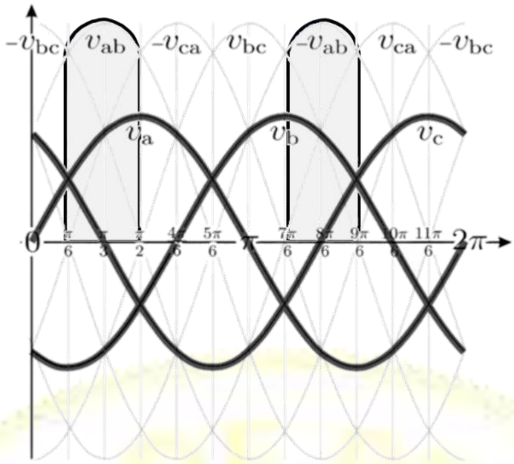
$$v_a = \sqrt{2}V_{ef}\sin(\omega t)$$
$$v_b = \sqrt{2}V_{ef}\sin\left(\omega t - \frac{2\pi}{3}\right)$$
$$v_c = \sqrt{2}V_{ef}\sin\left(\omega t + \frac{2\pi}{3}\right)$$

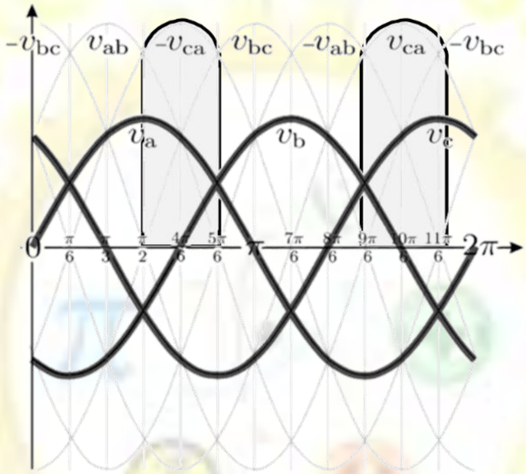


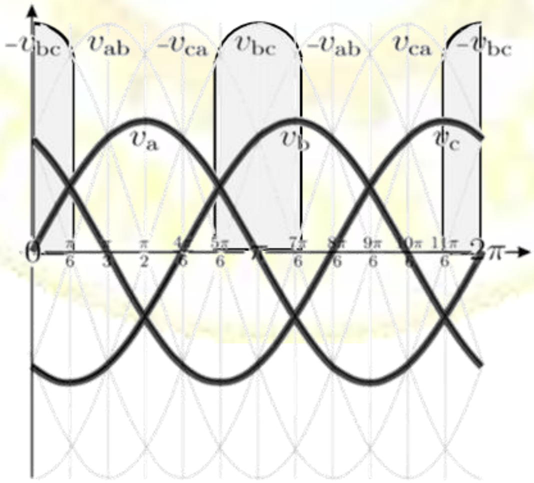






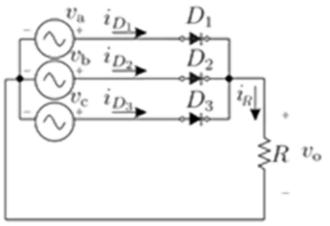

$-v_{bc}$ v_{ab} $-v_{ca}$ v_{bc} $-v_{ab}$ v_{ca} $-v_{bc}$
v_a v_b v_c
0
2π

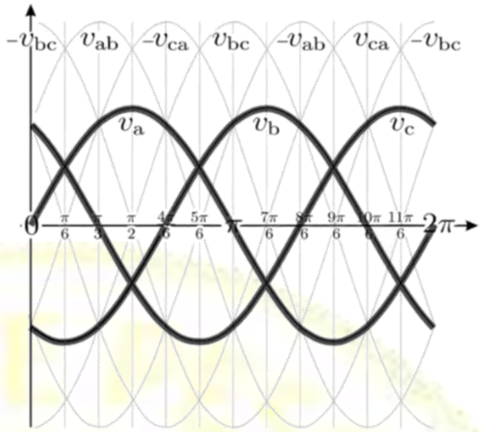

$-v_{bc}$ v_{ab} $-v_{ca}$ v_{bc} $-v_{ab}$ v_{ca} $-v_{bc}$
v_a v_b v_c
0
2π


$-v_{bc}$ v_{ab} $-v_{ca}$ v_{bc} $-v_{ab}$ v_{ca} $-v_{bc}$
v_a v_b v_c
0
2π

Retificador Trifásico de meia onda.

Relação entre as fases de 120^0





$$v_{max} = \sqrt{2}\, v_{ef}$$

$$v_a = \sqrt{2}\, v_{ef}\, \text{sen}(\varphi)$$

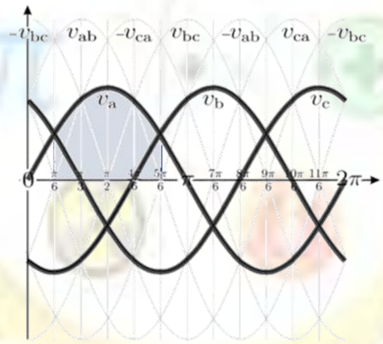
$$v_b = \sqrt{2}\, v_{ef}\, \text{sen}(\varphi - \frac{2\pi}{3})$$

$$v_b = \sqrt{2}\, v_{ef}\, \text{sen}(\varphi + \frac{2\pi}{3})$$

Tensão Média na Carga

$$\boldsymbol{Tenção\ de\ v_a\ na\ R\ (v_o)}$$

$$\boldsymbol{Tomando\ como\ referência\ o\ trecho\ \frac{\pi}{6}\ a\ \frac{5\pi}{6}}$$



$$\boldsymbol{v_a = \frac{1}{b-a}\int_a^b f(x)dx}$$

$$v_a = \frac{1}{\frac{5\pi}{6} - \frac{\pi}{6}}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} v_{Max} sen\ \varphi)d\varphi \qquad v_{Max} = \sqrt{2}V_{ef}$$

$$v_a = \frac{1}{\frac{2\pi}{3}}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} v_{Max} sen\ \varphi)d\varphi$$

$$v_a = \frac{3}{2\pi} v_{Max}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} sen\ \varphi)d\varphi$$

$$v_a = \frac{3}{2\pi} v_{Max} (-cos\ \varphi_{\frac{\pi}{6}}^{\frac{5\pi}{6}})$$

$$v_a = \frac{3}{2\pi} v_{Max} (-(-0{,}866 - (0{,}866))$$

$$v_a = \frac{3}{2\pi} v_{Max} (-(-1{,}732))$$

$$v_a = \frac{3}{2\pi} v_{Max} (1{,}732) \qquad 1{,}732 = \sqrt{3}\ e, v_{Max} = \sqrt{2} V_{ef}$$

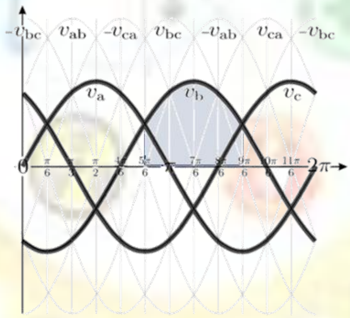
$logo$:

$$v_a = \frac{3\sqrt{2}.\sqrt{3}}{2\pi} v_{ef}$$

$$\boldsymbol{v_a = v_o = \frac{3\sqrt{6}}{2\pi} v_{ef}}$$

Tenção de v_b na R (v_o)

Tomando como referência o trecho $\frac{5\pi}{6}$ a $\frac{9\pi}{6}$



$$\boldsymbol{v_b = \frac{1}{b-a} \int_a^b f(x) dx}$$

$$v_b = \frac{1}{\frac{9\pi}{6} - \frac{5\pi}{6}} \int_{\frac{5\pi}{6}}^{\frac{9\pi}{6}} v_{Max} sen\ \varphi - \frac{2\pi}{3}) d\varphi \qquad v_{Max} = \sqrt{2} V_{ef}$$

$$v_b = \frac{1}{\frac{2\pi}{3}} \int_{\frac{5\pi}{6}}^{\frac{9\pi}{6}} v_{Max} sen\ \varphi - \frac{2\pi}{3}) d\varphi$$

$$v_b = \frac{3}{2\pi} v_{Max} \int_{\frac{5\pi}{6}}^{\frac{9\pi}{6}} sen\ \varphi - \frac{2\pi}{3}) d\varphi$$

$$v_b = \frac{3}{2\pi} v_{Max}(-cos\,(\varphi - \frac{2\pi}{3})_{\frac{5\pi}{6}}^{\frac{9\pi}{6}})$$

$$v_b = \frac{3}{2\pi} v_{Max}(-\left(cos\frac{9\pi}{6} - \frac{2\pi}{3}\right) - \left(cos\frac{5\pi}{6} - \frac{2\pi}{3}\right)$$

$$v_b = \frac{3}{2\pi} v_{Max}(-(\left(cos\frac{5\pi}{6}\right) - \left(\cos - \frac{\pi}{6}\right))$$

$$v_b = \frac{3}{2\pi} v_{Max}(-(-0{,}866 - (0{,}866\,))$$

$$v_b = \frac{3}{2\pi} v_{Max}(-(-1{,}732))$$

$$v_b = \frac{3}{2\pi} v_{Max}(1{,}732) \qquad 1{,}732 = \sqrt{3}\, e, v_{Max} = \sqrt{2}V_{ef}$$

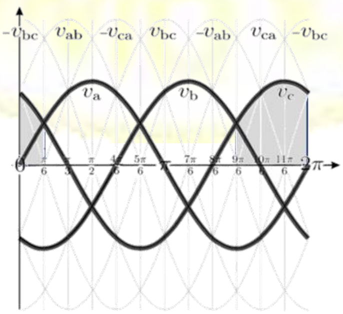
$logo$:

$$v_b = \frac{3\sqrt{2}.\sqrt{3}}{2\pi} v_{ef}$$

$$\boldsymbol{v_b = v_o = \frac{3\sqrt{6}}{2\pi} v_{ef}}$$

Tenção de v_c na R (v_o)

Tomando como referência o trecho $\frac{9\pi}{6}$ a $\frac{13\pi}{6}$ ($2\pi + \frac{\pi}{6}$, área que antecedeu v_a)



$$\boldsymbol{v_c = \frac{1}{b-a}\int_a^b f(x)dx}$$

$$v_c = \frac{1}{\frac{13\pi}{6} - \frac{9\pi}{6}} \int_{\frac{9\pi}{6}}^{\frac{13\pi}{6}} v_{Max} sen\ \varphi) d\varphi \qquad v_{Max} = \sqrt{2} V_{ef}$$

$$v_c = \frac{1}{\frac{2\pi}{3}} \int_{\frac{9\pi}{6}}^{\frac{13\pi}{6}} v_{Max} sen\ \varphi + \frac{2\pi}{3}) d\varphi$$

$$v_c = \frac{3}{2\pi} v_{Max} \int_{\frac{9\pi}{6}}^{\frac{13\pi}{6}} sen\ \varphi + \frac{2\pi}{3}) d\varphi$$

$$v_c = \frac{3}{2\pi} v_{Max} (-cos\ (\varphi + \frac{2\pi}{3})_{\frac{9\pi}{6}}^{\frac{13\pi}{6}})$$

$$v_c = \frac{3}{2\pi} v_{Max} (-\left(cos \frac{13\pi}{6} - \frac{2\pi}{3}\right) - \left(cos \frac{9\pi}{6} - \frac{2\pi}{3}\right)$$

$$v_c = \frac{3}{2\pi} v_{Max} (-(\left(cos \frac{9\pi}{6}\right) - \left(cos\ \frac{5\pi}{6}\right))$$

$$v_c = \frac{3}{2\pi} v_{Max} (-(-0{,}866 - (0{,}866\ ))$$

$$v_c = \frac{3}{2\pi} v_{Max} (-(-1{,}732))$$

$$v_c = \frac{3}{2\pi} v_{Max} (1{,}732) \qquad 1{,}732 = \sqrt{3}\ e, v_{Max} = \sqrt{2} V_{ef}$$

logo:

$$v_c = \frac{3\sqrt{2}.\sqrt{3}}{2\pi} v_{ef}$$

$$\boldsymbol{v_c = v_o = \frac{3\sqrt{6}}{2\pi} v_{ef}}$$

Potência Ativa

$$\boldsymbol{P = \frac{1}{b-a} \int_a^b p(\varphi) d\varphi}$$

$$P(\varphi) = \frac{v_{o.}(\varphi)^2}{R}\ d\varphi$$

$$P = \frac{1}{\frac{2\pi}{3}} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(V_{Max} sen(\varphi))^2}{R} d\varphi$$

$$P = \frac{3}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(\sqrt{2}V_{ef})^2(sen^2(\varphi)}{R}d\varphi$$

$$P = \frac{3.2.V_{ef}{}^2}{2\pi R}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} sen^2\, d\varphi$$

$$\boldsymbol{sin^2(\varphi) = \frac{1}{2}(1 - cos(2\varphi)}$$

$$P = \frac{6V_{ef}{}^2}{2\pi R}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1 - \cos(2\varphi)\, d\varphi \qquad u = 2\varphi_{d\varphi=\frac{du}{2}}^{du=2d\varphi}$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\varphi\, d\varphi - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}cos(\boldsymbol{u})\frac{\boldsymbol{du}}{\boldsymbol{2}})$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{1}{2}\left(\varphi_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\right) - \frac{1}{2}\,(sen(\boldsymbol{u}))\,\frac{\boldsymbol{1}}{\boldsymbol{2}})|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{1}{2}\left(\varphi_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\right) - \frac{1}{2}\,(sen(2\varphi))\,\frac{1}{2}|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) - \frac{\boldsymbol{1}}{\boldsymbol{4}}(sen(2\varphi)_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{1}{2}\left(\frac{4\pi}{6}\right) - \frac{1}{4}(sen\left(2\frac{5\pi}{6}\right) - \; sen\left(2\frac{\pi}{6}\right))$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\left(\frac{4\pi}{12}\right) - \frac{1}{4}(-0{,}866 - (0{,}866)$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\left(\frac{4\pi}{12}\right) - \frac{1}{4}\,(-\,1{,}732)$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\left(\frac{4\pi}{12}\right) - \frac{1}{4}\,(-\,\sqrt{3})$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\left(\frac{4\pi}{12}\right) + (\frac{\sqrt{3}}{4}))$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\left(\frac{\pi}{3}\right) + (\frac{\sqrt{3}}{4}))$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{4\pi}{12} + \frac{3\sqrt{3}}{12})$$

$$P = \frac{3V_{ef}{}^2}{\pi R}(\frac{1}{12}(4\pi + 3\sqrt{3}))$$

$$P = \frac{V_{ef}{}^2}{\pi R}(\frac{1}{4}(4\pi + 3\sqrt{3}))$$

$$\boldsymbol{P = \frac{V_{ef}{}^2}{4\pi R}(4\pi + 3\sqrt{3})}$$